OCR A Physics A level

Topic 6.2: Electric Fields

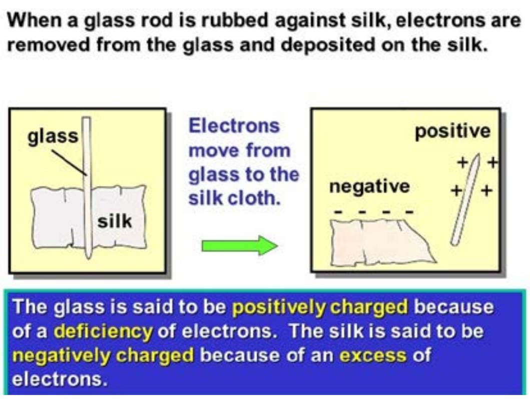
(Content in italics is not mentioned specifically in the course specification but is nevertheless topical, relevant and possibly examinable)

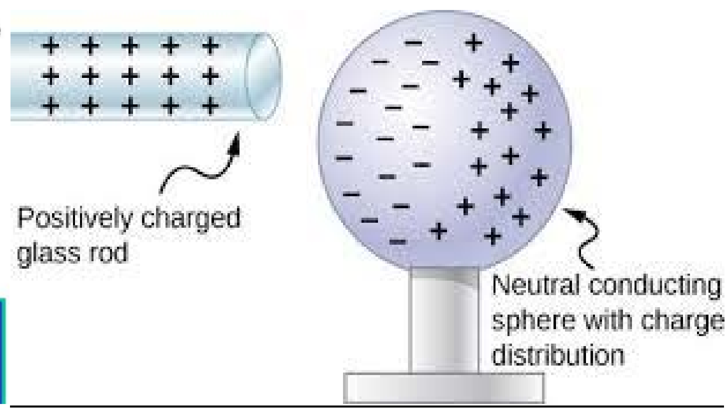
Definition of the Electric Field

An electric field is a region of space in which charged particles are subject to an electrostatic force.

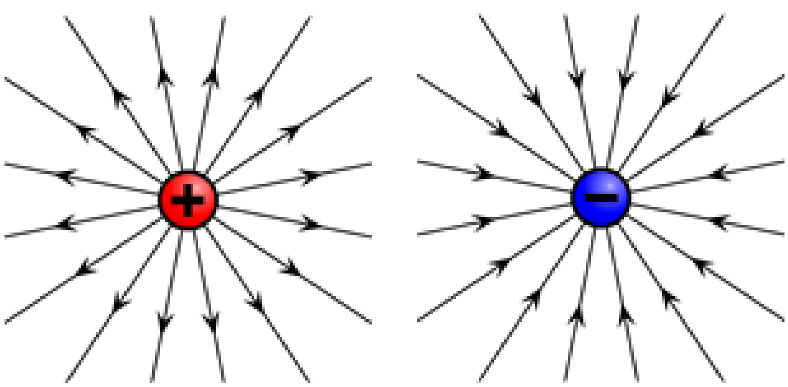
Static Electricity and Induced Charges

These forces can be shown to exist by generating a static charge on an object, for example, rubbing a glass rod with a silk cloth. The glass rod will lose electrons to the silk cloth and so become slightly positive. This rod can then be used to induce dipole charges and attract small neutral objects, such as pieces of paper or water from a tap. When the positive rod is close to a neutral object, the electric field due to the rod will attract electrons in the object and shift their positions to be closer to the rod. This causes the object to be more negative on the side closest to the rod and more positive on the opposite side i.e. the object remains neutral overall but becomes polar. The rod and the induced charge on the object will then attract.





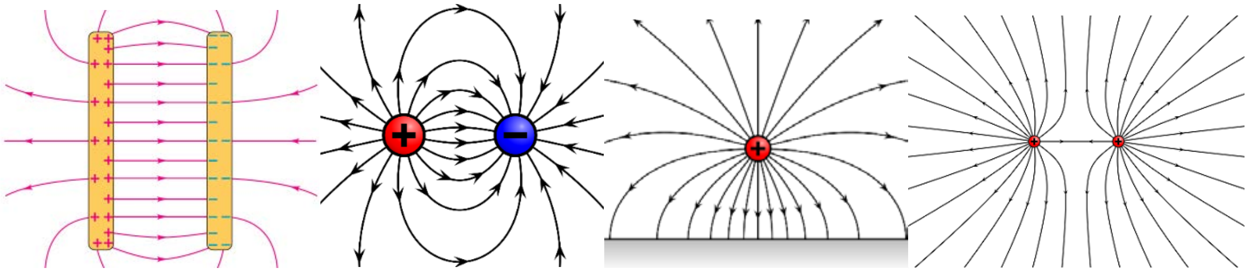
Electric Fields of Point Charges and Charged Spheres



Protons and electrons are charged particles that can be modelled as point charges i.e. their charge exists at a single point in space. Point charges have radial fields which look like the spokes of a wheel. Electric field lines point outwards from a positive charge or inwards towards a negative charge as the direction of the field represents the direction of the electrostatic force on a positive charge at that point. The strength of the field decreases with the radial distance or the distance away from the charge. This is shown by the decreasing density of the field lines, the less tightly packed the field lines are the weaker the electric field is at that point and the weaker the electrostatic force will be on a given charge.

Spherically charged spheres, e.g. an ion or the metal sphere used in a Van de Graaff generator, can also be modelled as point charges as outside the sphere the electric field is radial and decreases in strength with distance away from the surface of the sphere.

Electric Field Lines



The diagrams above show the electric field lines around positive and negative point charges. Field lines generally point from positive to negative, are equally spaced as they exit a surface and are perpendicular to the surface. Lines that stretch from a positive charge to a negative charge show attraction whereas field lines that do not join up identify repulsion. Here are some other common geometries.

Electric Field Strength

The strength of an electric field, E is defined as the force, F applied per unit charge Q on an object and is expressed as:

$$E = \frac{F}{Q}$$

An E field has the unit NC^{-1} or Vm^{-1} (these units are interchangeable) and is a vector quantity as it has a direction, the direction a positive charge would accelerate at that point, and magnitude, the size of the force experienced at that point per unit charge.

Forces between Point Charges

Coulomb's law states that any two point charges exert an electrostatic force between them that is proportional to the product of their charges and inversely proportional to the square of the distance between them. This is similar to Newton's law of Gravitation (see topic 5.5) except that the charges are replaced by masses and that gravity is a solely attractive force hence the negative sign in the equation for the gravitational force.

Coulomb's law mathematically states that for two charges Q and q at a distance r apart

$$F \propto Qq \text{ and } F \propto \frac{1}{r^2}$$

Therefore, combining these statements and determining the constant of proportionality experimentally we reach

$$F = k\frac{Qq}{r^2} \text{ where } k = \frac{1}{4\pi\varepsilon_0} = 8.99 \times 10^9 \text{ Nm}^2\text{C}^{-2}$$

where ε_0 is the permittivity of free space (8.85 x 10^{-12} Fm^{-1}). Permittivity is a constant that defines the ability of a material to become polarized and store charge. The permittivity of free space is the permittivity of a vacuum and as there is no matter in a vacuum to become polarized this is the lowest value possible. In matter generally, the permittivity can be written as a constant, the relative permittivity, ε_r, multiplied by the permittivity of free space.

Electric Field of a Point Charge

Using Coulomb's law and the formula for electric field strength, it is possible to form an expression for the electric field of a point charge. To do this we must imagine bringing a small test charge of charge, q, towards a large point charge of charge, Q, such that the effect of the test charge on the point charge is negligible i.e. the test charge does not cause the point charge to move.

The electric field strength on the test charge can be written as

$$E = \frac{F}{q}$$

Therefore, using Coulomb's law for the force

$$E = \frac{Q}{4\pi r^2 \varepsilon_0}$$

Gravitational and Electric Fields

To explore the concept of a field it is useful to draw similarities between gravitational and electric fields.

	Gravitational	Electrical
Property that creates the Field	Mass	Charge
Type of Field	Always attractive (the arrows on the field lines always point inwards towards the mass)	+ ve charge = repulsive to positive charges - ve charge = attractive to positive charges
Field Strength	Force per unit mass $g = \frac{F}{m}$	Force per unit positive charge $E = \frac{F}{q}$
Forces between Particles	$F \propto Mm, F \propto \frac{1}{r^2}$	$F \propto Qq, F \propto \frac{1}{r^2}$
Force and Field Strength	$F = -G\frac{Mm}{r^2}$ $g = -G\frac{M}{r^2}$	$F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}$ $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$
Shape of Field around a Point	Point mass produces radial field	Point charge produces radial field

Uniform electric fields

A uniform field is a field which has no dependence upon position i.e. the field is constant in space. A uniform electric field is produced between two parallel oppositely charged plates. A particle between such plates is subject to a constant force no matter where it resides between the plates.

Work Done and Electric Potential

Work done is a measure of the energy transferred to an object by a force. It is the product of the force and the distance over which the force acts in the direction of the force.

$$W = Fd$$

Therefore, work is done when moving a charge in an electric field. The work done per unit charge in moving a charge from one point to another in the field defines the potential difference between those two points.

$$V = \frac{W}{Q}$$

The **electric potential** at a single point can be defined as the **work done per unit charge to bring a positive charge from infinity to that point**. The electric potential at infinity is often defined as 0. Hence, the potential difference between two points A and B is the difference between the electric potential at B, V_B, and the electric potential at A, V_A i.e. $V = V_B - V_A$.

As the electric field can be defined by

$$E = \frac{F}{Q}$$

Then

$$W = VQ = Fd = EQd$$

This implies $V = Ed$, however the electric field must be constant over the distance the charge moves therefore we cannot apply this equation to non-uniform fields. In this case of parallel plates, electric field strength can be defined as $E = \frac{V}{d}$ where V is the potential difference applied across the plates and d is the distance between them. It is clear to see from this equation that the field can be defined in units of Vm^{-1}.

Motion of a Charged Particle in a Uniform Electric Field

Due to the similarities of the gravitational and electric fields it is possible to model the motion of a charged particle through an electric field as similar to that of a massive particle through a gravitational field i.e. **projectile motion**.

*In **Millikan's oil drop experiment**, oil droplets are charged with a small electric charge and allowed to fall under gravity in a **vertical uniform electric field**. The electric field is generated by two oppositely charged parallel plates to ensure that it is uniform. As these drops fall, the voltage between the plates can be increased and to produce a larger electric field in the gap. This produces an upward acting force which can be increased to balance with the force of gravity. When the oil drops hover in midair, we can deduce that the **weight force downwards must be equal to the force of the electric field upwards** as there is no acceleration.*

$$mg = EQ = \frac{VQ}{d}$$

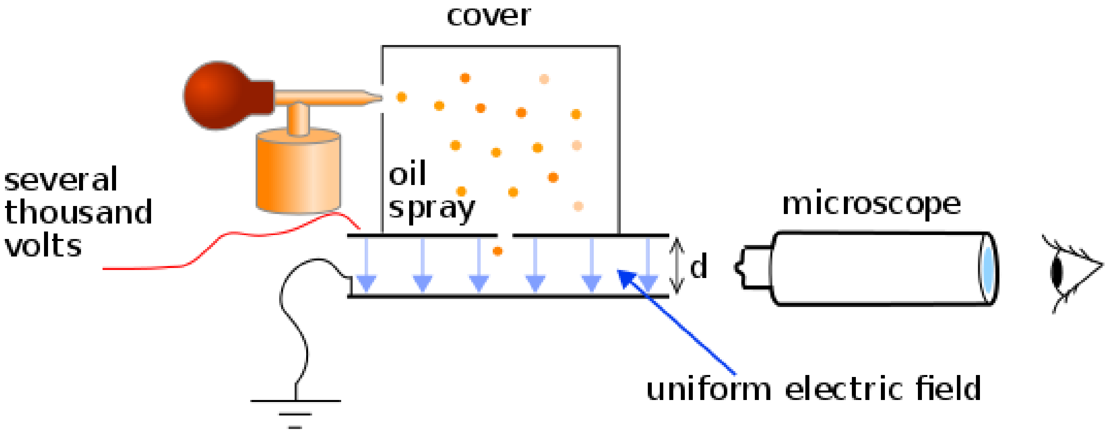
The mass and voltage between the plates are known and the charge on the drop can be quantized as $Q = -ne$ i.e. that the charge is a whole number, n, of electrons gained by the oil drop. Therefore, a specific droplet of charge $-ne$ will be stopped by a voltage, V, given by this equation

$$V = \frac{mgd}{ne}$$

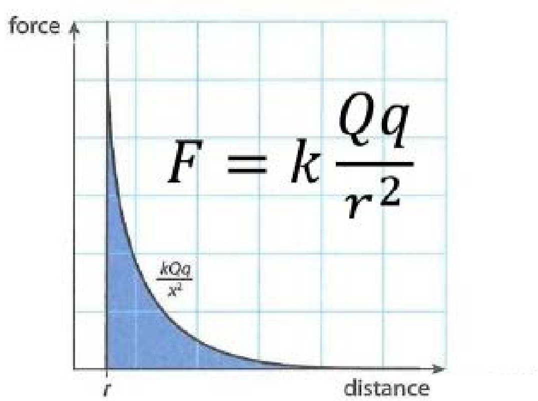
Therefore, only certain values for the voltage will be measured as only small negative integer charges are produced on the droplets. By repeating the experiment for many *droplets, Millikan confirmed that the charges were all small integer multiples of a certain base value:*

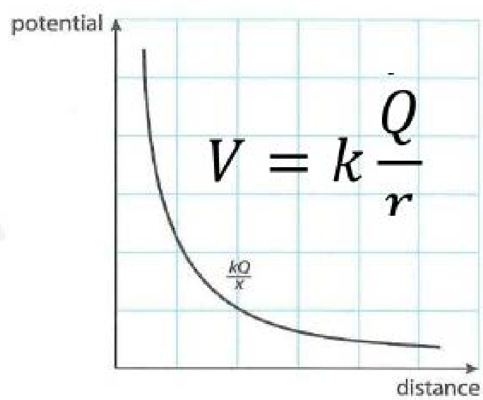
$$e = 1.6022 \times 10^{-19} C$$

Thus, Millikan's oil drop experiment allowed for an accurate determination of the elementary charge.



Work done on Point Charges





Work is done when a charge, q, is brought in from infinity to a distance r from a point charge, Q. This work is the electrical energy of this geometry. The graph on the left shows the force needed as a function of the distance between the particle and the point charge. The work is the area underneath this graph from infinite separation to the distance between the charges, r.

If the charges Q and q are oppositely charged then the electrical energy would be negative as kinetic energy would be gained q by the as it accelerates towards the charge Q from infinity. This means that it would take energy to separate the particles instead and the value of the potential energy would be how much energy it would take to move the particle q from a distance r away from the Q to infinity.

The electrical energy can be defined by the following.

Work done by a force over a distance is given by

$$W = Fd$$

However Coulomb's law states that

$$F = \frac{Qq}{4\pi r^2 \varepsilon_0}$$

The work done and so the electrical potential energy needed to move a point charge q from infinity to a radius r is given by

$$E_e = \frac{Qq}{4\pi r \varepsilon_0}$$

with the electrical energy defined as E_e to distinguish it from the electric field.

As the electric potential is defined as the electrical potential energy per unit charge

$$V = \frac{E_e}{q}$$

Therefore

$$V = \frac{Q}{4\pi r \varepsilon_0}$$

which is shown in the graph on the right above.